Journal of Engineering Physics and Thermophysics, Vol. 69, No. 6, 1996

SOME REGULARITIES AND MODELS IN HEAT TRANSFER THEORY

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UDC 621.438:536.2

*The results reported pertain mainly to the classical or "fundamental" subdivisions in the theory of heat and mass transfer. They correspond to the laminar mode of flow, which is considered to be a subject explored in detail owing to advances in computational mathematics. The laminar mode has turned out to be most important and interesting in creating a new generation of gas-turbine power plants distinguished by high efficiency (~ 60%) and specific power per unit flow rate of air (~ 1 MJ/kg). The thesis is confirmed that applied scientific-engineering developments give a powerful impetus to the formulation of fundamental studies and often determine the trend of the search for physical regularities. Being confined by the framework of a plenary report, we could not touch on all the aspects of the studies devoted to gas-turbine plants conducted at the Institute of High Temperatures of the Russian Academy of Sciences. Some of the results obtained have been published [1-5], while others another are being prepared for publication.**

It is known that in rotary heat engines, such as gas turbines, the flow around nozzles and blades is characterized by a high degree of turbulence of the oncoming flow. Figure 1 shows experimental data [6] indicating that an increase in the turbulence level Tu to 3–8% changes both the intensity of heat transfer (or the Nusselt number) and the dependence Nu = f(Re). When Nu = $0.296(\mathrm{Re}_x)^{0.5}$ for laminar flow over a plate, an increase in the turbulence of the oncoming flow causes an increase in the exponent of the Reynolds number to 0.65 (at Tu ≈ 10%) [6].

Results of experimental studies of the internal structure of a "pseudolaminar" boundary layer have revealed velocity fluctuations that are generated in it. This is confirmed by the reduced extent of the laminar zone of the boundary layer with rise in the degree of turbulence of the oncoming flow [5].

Obviously, it is impossible to describe the full diversity of heat transfer phenomena in turbulized flows using only the degree of turbulence. From the practical viewpoint, in particular, in studying heat transfer on the surfaces of gas turbine blades, the nonproportional variation of the heat flux in different zones of the boundary layer is important.

Figure 2 presents a comparison of heat load distributions along a blade surface at low and elevated degrees of turbulence [7]. The hatched and blackened regions are those of increased and decreased heat fluxes with Re = $1.67 \cdot 10^6$ and variations of the degree of turbulence in the range of Tu = 0.2–4.0%. As the Re number decreases, the picture remains unchanged though less pronounced. The point of transition of the boundary layer from laminar to turbulent flow is distinctly seen when the heat flux abruptly increases as it moves along the windward or leeward (the blackened sections) surface of the blade. At a higher degree of turbulence, the transition point is displaced toward the leading edge. As for the heat transfer intensity, the zone of the laminar boundary layer is most sensitive to an increase in the degree of turbulence Tu, while in the case of the turbulent mode this influence is not great.

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 69, No. 6, pp. 885-896, November-December, 1996. Original article submitted July 12, 1996.

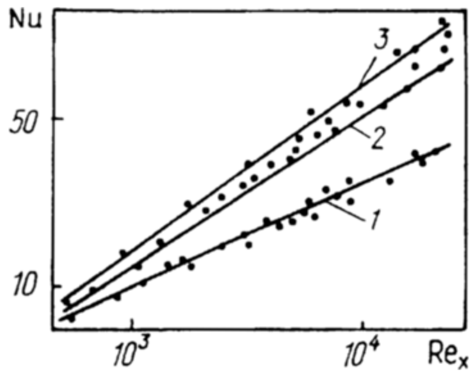


Fig. 1. Typical change of the local heat transfer (Nu) along a plate (Re) for different levels of turbulence of the oncoming flow: 1) Tu = 0.2%, 2) 3, 3) 8.

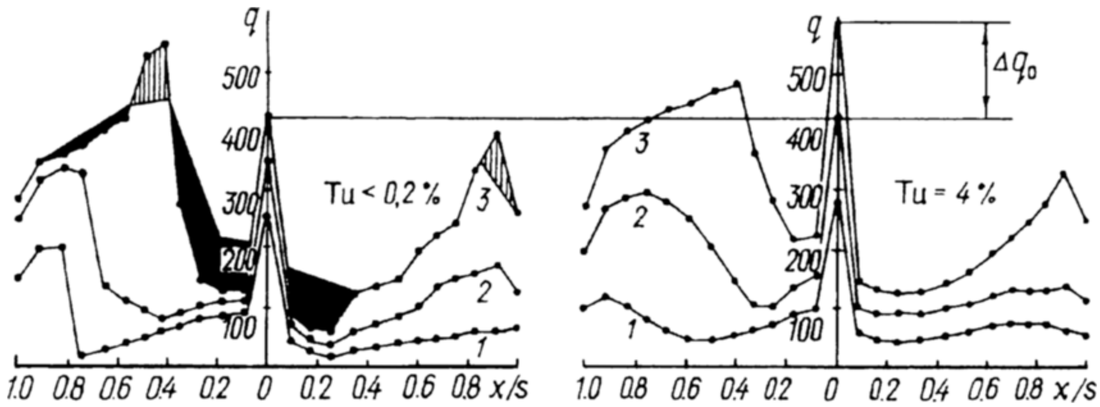


Fig. 2. Influence of the Reynolds number and the degree of turbulence on the distribution of the heat transfer rate along a gas turbine blade: 1) Re = $0.56 \cdot 10^6$, 2) $1.1 \cdot 10^6$, 3) $1.67 \cdot 10^6$. q, kW/m^2.

It is of interest to note that with increase in Tu, the typical "throwing away" of the heat flux at the end of the transient regime of the flow ceases to be observed.

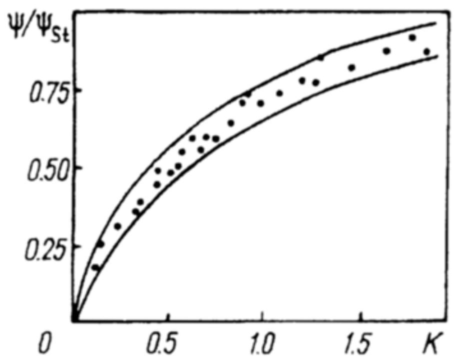
Evaluation of the experimental results represented in Fig. 2 allows us to assert that the turbulence of the oncoming flow makes the heat load distribution along both blade surfaces of a gas turbine smoother. This fact facilitates the design of high-potential blades. However, in the vicinity of the leading edge (in the region of laminar flow), an increase in the degree of turbulence Tu has an adverse effect on the thermal regime and, as a consequence, on the increase in the required flow rate of the coolant. This finding fosters the development of new physical models of the laminar boundary layer in turbulized flows.

Until recently it was believed that the influence of the degree of turbulence of an oncoming flow can be accounted for by a simple change in the liquid viscosity. Thus, the notion of the effective viscosity $\eta_\Sigma = \eta + \eta_u$ is introduced, where η is the molecular viscosity, while η_u is the additional or apparent viscosity, which depends on the degree of turbulence Tu [8]:

$$\eta_u = 0.164\, y \mathrm{Tu} V_\infty . \tag{1}$$

Here y is the transverse coordinate reckoned from the surface of the body with a flow around it; V_∞ is the velocity of the oncoming flow.

It is easy to verify that the model described in [8] is based on the same ideas that have been employed by Prandtl to describe a turbulent boundary layer. However, adding the term η_u to the effective viscosity η_Σ does not allow full agreement to be reached between experimental and calculated data on heat transfer in the laminar



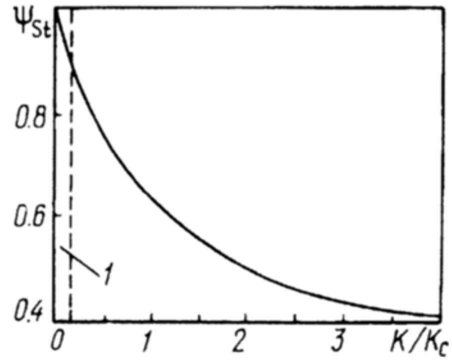


Fig. 3. Ratio of the criteria of thermal efficiency Ψ and Ψ_{St} vs the dimensionless parameter K.

Fig. 4. Stabilized criterion of thermal efficiency Ψ_{St} vs the ratio K/K_c (for $K > 4$): 1) range of the parameters investigated previously.

boundary layer. In [7] it is established that there are geometries of gas turbine blades on which the heat flux near the leading edge remains almost unchanged with increase in Tu.

To analyze the paradox observed in experiments [7], we have employed a heat transfer model developed previously for conditions of dust-laden flows around bodies [4].

If we assume that a turbulent mole generated far from the body is incorporated into the laminar boundary layer as some whole, then at each moment it behaves like a solid rotating particle moving relative to the remaining liquid with a velocity V' equal to the fluctuation velocity component, $V' = V(t) - \overline{V}$. This principle of instantaneous "hardening" makes it possible to visualize seemingly the mechanism of the influence of external turbulence and, as a minimum, to provide a general representation of the similarity criterion.

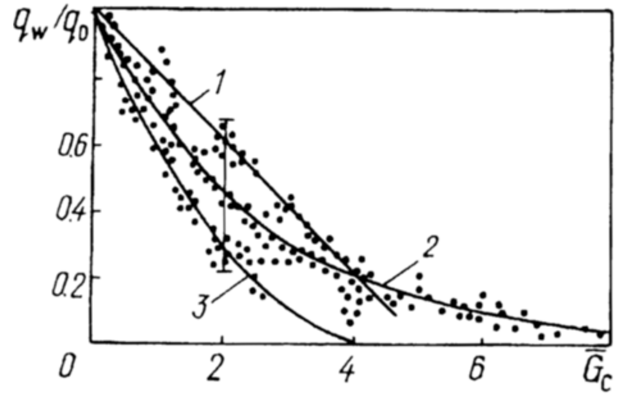
The deeper the mole penetrates into the boundary layer, the greater, obviously, its influence on convective heat transfer. The penetration of the mole is blocked by a longitudinal stream of the averaged flow with the velocity $U_e(x) = \beta x$, where $\beta = (dU_e/dx)_{x=0} \approx \varphi_N/R_N$ with φ_N being the form factor and R_N the characteristic dimension of frontal bluntness. The real trajectory of the mole will most likely to be curvilinear and dependent on its scale. We can introduce, as a similarity criterion, the dimensionless ratio of the penetration time of the turbulent mole $\tau_{Tu} \approx \delta/V'$ to the characteristic time of hydrodynamic transfer in the averaged flow $\tau_U \approx 1/\beta = 1/(dU_e/dx)_0$:

$$C_t = \frac{\tau_U}{\tau_{Tu}} = \frac{V'}{\beta\delta} = \frac{\rho V'}{A\sqrt{\beta(\eta\rho)}} \approx \frac{\rho V'}{(\alpha/c_p)_0}. \tag{2}$$

Here $\delta = Ax/\sqrt{\mathrm{Re}}$ is the thickness of the boundary layer in the vicinity of the stagnation point, the heat transfer coefficient $(\alpha/c_p)_0$ is calculated at Tu ≈ 0, and A is a proportionality constant.

The similarity criterion C_t is proportional, with accuracy to a constant, to the criterion β_t introduced in [9]. Although its physical interpretation does not seem to be sufficiently convincing, it is important that the extensive experimental data collected in [9] now allow a new interpretation of the presence or absence of an increase in the heat load on the blades of gas turbines.

As in the heat transfer model for dust-laden flows [4], the shape and dimensions of a body with a flow around it are mainly manifested through the velocity gradient $\beta = (dU_e/dx)_0$. The greater the value of β, the higher the probability of carrying particles or turbulent moles out of the vicinity of the frontal point. Correspondingly, their influence on convective heat transfer is weaker, with other conditions being equal. On the blades used in the experiments in [7], β changed by almost a factor of three. For a blade geometry for which β was high, the influence of external turbulence did not manifest itself, while in the case of the minimum β, it was more than 40% at Tu = 4%.



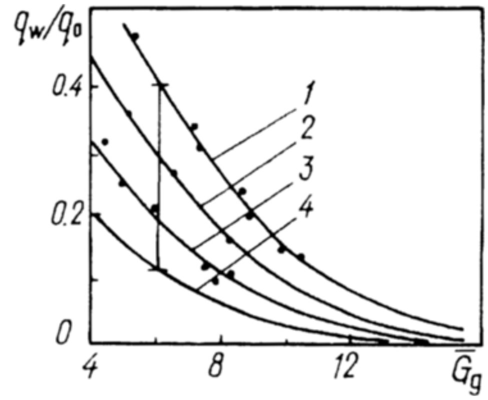


Fig. 5. Influence of coolant injection at the flow rate $\overline{G}_c$ on the relative intensity of heat transfer in a turbulent boundary layer: 1, 2, 3) calculated relations; points – experimental data.

Fig. 6. Influence of coolant injection and surface porosity Π_w on the relative intensity of heat transfer: 1) $\Pi_w = 0.03$, 2) 0.12, 3) 0.36, 4) 0.45.

Qualitative agreement between experiment and the model does not allow so far an exact prediction of the optimum blade shape. Further investigations are needed, first of all, for determination of the influence of the turbulence scale [10].

We now pass to another problem concerned with thermal protection. In [2] it is shown that substitution of direct-flow for convective cooling of the blades of gas turbines will result in a considerable decrease in coolant consumption and temperature nonuniformity inside the blade. However, the small thickness of the porous wall raises the question about the reliability of the available models of conjugate heat transfer, in particular, about the reliability of experimental data on the injection effect in laminar and turbulent boundary layers and on the value of the coefficient of internal heat transfer α_v between the filtering coolant and the porous matrix.

Discussion of these problems at the seminar supervised by A. I. Leont'ev, Academician of the Russian Academy of Sciences (RAS), has brought researchers of the Institute of High Temperatures of the RAS to the need to develop a new heat-mass transfer model for a porous medium. Its radical difference from all known models lies in the fact that the thermal conductivity of not only the matrix λ_s but also the coolant λ_c is taken into account. Moreover, it is postulated that on the outer (heated) side of the porous envelope the coolant temperature $T_c|_{y=h} = T_{cw}$ and the matrix temperature $T_s|_{y=h} = T_{sw}$. are equal: $T_{cw} = T_{sw}$. Only with such an assumption made, can one avoid taking into account a temperature unevenness in the boundary layer whose longitudinal dimensions are smaller than the thickness of the boundary layer and whose amplitude is commensurable with the temperature drop in this layer. For a steady-state thermal regime inside the permeable wall two temperature profiles are formed, namely, $T_s(y)$ for the solid porous matrix and $T_c(y)$ for the filtered-coolant flow. They are described by the following system of equations:

$$\lambda_s (1 - \Pi) \frac{d^2 T_s}{dy} = \alpha_v (T_s - T_c) , \quad - \lambda_c \Pi \frac{d^2 T_c}{dy^2} + c_{pc} G_c \frac{dT_c}{dy} = \alpha_v (T_s - T_c) . \tag{3}$$

Here Π is the porosity; α_v is the coefficient of volumetric internal heat transfer; c_{pc}, G_c are the heat capacity and flow rate of the coolant, respectively.

The boundary conditions at the inlet to the porous matrix ($y = 0$) allow convective heat transfer to proceed between the coolant and the first layer of fibers forming the porous skeleton:

$$\lambda_s \left. \frac{dT_s}{dy} \right|_{y=0} = \mathrm{St}_{en} G_c c_{pc} (T_s - T_c)_{y=0} . \tag{4}$$

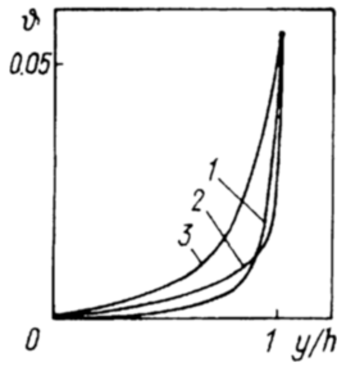


Fig. 7. Dimensionless-temperature distribution in a system of porous matrix-coolant: 1) one-temperature model $\vartheta_s = \vartheta_c$, 2) ϑ_c, 3) ϑ_s.

The boundary conditions at the outlet from the porous wall take into account the injection effect – a decrease in convective heat transfer upon supplying coolant to the boundary layer:

$$\frac{q_w}{q_0} = 1 - \gamma b_t . \qquad (5)$$

Here q_0 is the heat flux on the impermeable wall; $b_t = G_c/(\alpha/c_p)_0 = G_c/[\rho_c U_e \mathrm{St}_0]$.

An analytical solution to system of equations (3) is obtained that makes it possible to calculate the temperature difference between the coolant and the matrix at any point of the impermeable wall $(T_s - T_c)$ and, consequently, to determine the integral amount of heat consumed by the coolant upon filtration through the permeable matrix. It is convenient to introduce the dimensionless efficiency of direct-flow cooling in the form

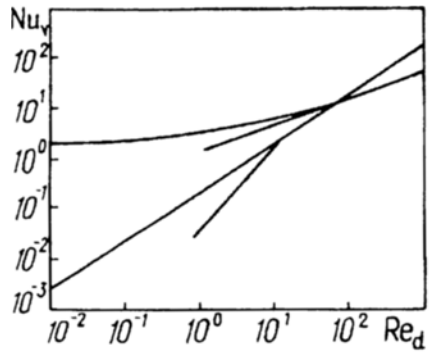
$$\Psi = \frac{\int_0^h \alpha_v (T_s - T_c)\, dy}{q_w} .$$

It is established that Ψ is a function of the following three dimensionless parameters:

$$R = \mathrm{Re}_d \mathrm{Pr}_c \left(\frac{h}{d}\right), \quad K_c = \frac{\alpha_v h}{G_c c_{pc}} \quad \text{and} \quad K = R \frac{\lambda_c \Pi}{\lambda_s (1 - \Pi)} .$$

Here $\mathrm{Re}_d = G_c d/\eta_c$, $\mathrm{Pr}_c = \eta_c c_{pc}/\lambda_c$, η_c, c_{pc}, λ_c are the viscosity, heat capacity, and thermal conductivity of the coolant, respectively. At $K \le 4.0$ the efficiency Ψ depends on the thickness of the porous wall, more exactly, on the ratio h/d (Fig. 3). At high values of K the criterion Ψ becomes stabilized in the sense that a further increase in h does not cause a change in it; however Ψ_{St} depends on the ratio K/K_c (Fig. 4). The threshold value $K = 4.0$ corresponds to the circumstance that at the inlet to the permeable wall the coolant temperature $T_c|_{y=0}$ and that of the matrix $T_s|_{y=0}$ are rather close to the initial coolant temperature $T_{c\infty}$, and therefore the heat transfer at the inlet does not exert a substantial influence on the thermal regime of the entire porous envelope (i.e., St_{en} ceases to be one of the governing criteria).

The solution obtained makes it possible to expalin well-known contradictions of experimental data on both the injection effect and dimensionless correlations for internal heat transfer of the type $\mathrm{Nu}_v = f(\mathrm{Re}_d; \mathrm{Pr}_c)$. Figure 5 taken from [11] illustrates the effect of injection into a turbulent boundary layer. As is seen, the experimental data on the heat flux on a porous surface q_w versus the dimensionless flow rate of the injected gas $\overline{G_c} = b_t$ show a large scatter. The calculated curves also differ markedly. Curve 1 corresponds to the recommendation given in [12, 13]. Curve 2 is of an exponential nature, i.e., it tends asymptotically to zero as $\overline{G_c} = b_t \to \infty$. It is described in detail in [14] and is basically consistent with the experimental data of [15]. Finally, curve 3 is calculated from the formula of the limiting law of [11]. Such a considerable scatter of experimental data does not allow, in essence, reliable evaluation of the technical parameters of a direct-flow cooling system. Therefore it is necessary to analyze



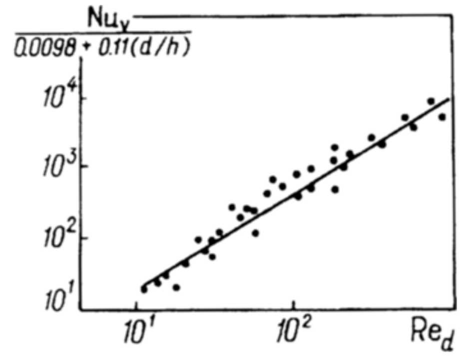


Fig. 8. Comparison of empirical dependences for internal heat transfer in porous structures. Curves – processed data of different authors [18].

Fig. 9. Influence of the dimensionless sample thickness on the criterial relation of internal heat transfer in porous envelopes [18]. Solid line – $\mathrm{Nu_v} = [0.0098 + 0.11(d/h)]\mathrm{Re}_d^{1/3}\mathrm{Pr}_c^{1/3}$; points – experimental data.

thoroughly the accuracy of measurements and possible reasons for the inadequacy of the models used in the experiments.

Figure 6 presents experimental results of [16]. Curves 2-4 are averaged results of measurements made on permeable models differing only in the size of the holes and the distances between them. By changing the degree of perforation, the author varied the relative area of the holes on the surface of the model from 0.36 (curve 3) to 0.12 (curve 2) and further to 0.03 (curve 1). Curve 4 illustrates the effect of injection through a porous surface with $\Pi_w = 0.45$. The curve is close to calculated curve 2 in Fig. 5. It is easy to see that at the same flow rate of the injected gas $\overline{G_c}$ the injection effect undergoes abrupt attenuation with increase in the degree of perforation.

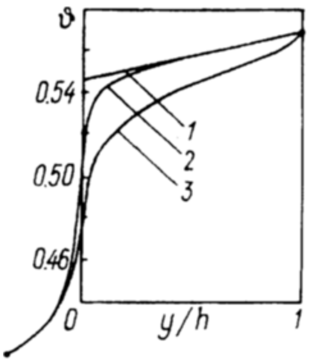
At a high injection rate it appears that individual jets, leaving the holes at a high speed, seemingly, "perforate" the boundary layer without causing substantial rearrangement of the temperature or velocity field in the near-wall zone. Hence it follows that perforated envelopes, the more so those with a large perforation step, cannot provide effective thermal protection.

Figure 7 presents calculated temperature profiles for a high filtration rate $b_t = \overline{G_c} = 3$: curve 2 pertains to the coolant temperature, and curves 3 to the temperature of the porous matrix. The thickness of the permeable wall amounts to only 1 mm; nevertheless, even this value was sufficient to produce the entire temperature drop in the coolant. Curve 1 corresponds to a calculation under thermal-equilibrium conditions, when the temperature distributions in the solid and gas phases coincide ($T_s \equiv T_g$).

As is seen, the degree of temperature nonuniformity in permeable envelopes with intense injection is rather high, and the temperature gradients at the external surface are great. In this situation it is hardly possible to evaluate heat fluxes q_w by measuring the temperature drop at the inlet and outlet of the porous wall, especially at high flow rates of the coolant. We believe that in comparing the data on the injection effect of different authors, preference should be given to data obtained on failing calorimeters (in particular, on sublimating materials). Only in this case is there no temperature nonuniformity both inside the calorimeter and on its surface.

Equally interesting conclusions can be drawn about the internal heat transfer in porous structures, in particular, if we look at numerous experimental data processed in the form of the criterial regularity $\mathrm{Nu} = c\mathrm{Re}^n$. Figures 8 and 9 taken from [17, 18] distinctly show two tendencies. The first tendency lies in the fact that in the range of low Reynolds numbers $\mathrm{Re}_d < 10$ the divergence of the experimental data increases with decreasing Re. The exponent n in the criterial law varies from 0 to 1.3, which exceeds considerably the range of n for tube flows. The second tendency is related to the scale effect (Fig. 9). It turns out that the intensity of the internal heat transfer depends on the thickness of the porous wall, more exactly, on the ratio h/d. Numerical calculations by the model described above allow certain conclusions to be made about the reasons for this situation.

We compare the results given in Figs. 10 and 11. Figure 10 shows a temperature profile in a porous matrix (curve 1) and two versions of a calculation of the temperature distribution in the coolant. First a calculation was



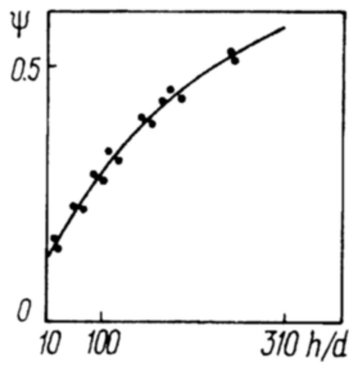


Fig. 10. Dimensionless-temperature profiles of a coolant for two laws of internal heat transfer: 1) matrix temperature; 2, 3) coolant temperatures.

Fig. 11. Criterion of thermal efficiency Ψ vs the dimensionless wall thickness.

made for a stabilized laminar flow in a tube (Nu = const, curve 2) and then for the empirically chosen regularity Nu = $0.1 Re_d Pr_c$ (curve 3). Despite a considerable difference in the coolant temperatures, both versions yield practically the same values of the thermal efficiency Ψ (Fig. 11). This criterion increases considerably with an increase in the thickness of the porous wall (h/d).

The scale effect corresponds approximately to a twofold increase in Ψ with increase in the dimensionless envelope (h/d) by a factor of four (Fig. 11). It is noteworthy that the thickness of the envelope was varied in almost the same range in the experiments whose results are shown in Fig. 9. The mechanism of the scale effect lies in the fact that at low Reynolds numbers ($Re_d \approx 1$) and an envelope thickness amounting to several millimeters, the criterion Ψ is considerably smaller than unity. This means that a smaller portion of the heat flux q_w is supplied to the coolant by heat transfer inside pores, while the contribution of the thermal conductivity of the coolant and the heat transfer at the inlet to the porous structure turns out to be the governing factor.

CONCLUSIONS

1. The two flow modes, i.e., laminar and turbulent, differ in the laws of change of the boundary layer thickness and heat transfer rate with increase in the Reynolds number.

2. A number of disturbing factors exist that lead to violation of the laminar heat transfer law. They include the increased turbulence of the oncoming flow and the presence of fine condensed particles (heterogeneous flows). A unified physical model is suggested for explanation of heat transfer enhancement in both cases.

3. An analysis of experimental data on external and internal heat transfer in direct-flow cooling systems is made. The reasons for the considerable disagreement among the published data are discussed using results of a calculation of the thermal regime of porous envelopes with a filtering coolant by a new model.

NOTATION

b_t, dimensionless flow rate of the coolant; c_p, specific heat; C_t, criterion determining the effect of turbulence; d, characteristic dimension of the pores; h, thickness of the permeable wall; L, length; Nu, Nusselt number; Re, Reynolds number; St, Stanton number; S, longitudinal coordinate; R, bluntness radius; T, temperature; Tu, degree of turbulence; (u, v), components of the velocity vector; (x, y), coordinates; α, heat transfer coefficient; β, velocity gradient; γ, injection coefficient; δ, thickness of the boundary layer; λ, thermal conductivity; η, viscosity; ρ, density; τ, time; ϑ, dimensionless temperature; Ψ, criterion of thermal efficiency for direct-flow cooling; c, proportionality constant.

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